

## Lesson 22

Tuesday, June 01, 2010  
1:47 PM

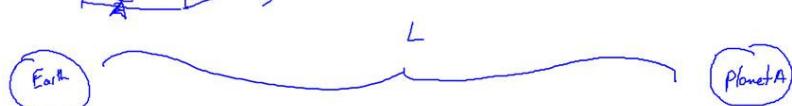
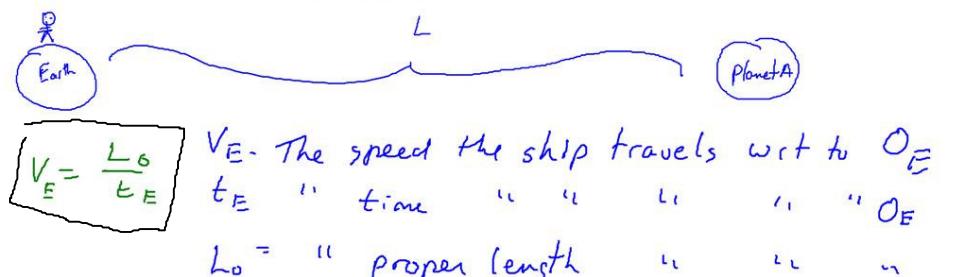
### Length Contraction

if the velocity of light is absolute,  $\therefore$  distance and time is not

2 cases to consider

We want to send a space vehicle the distance from Earth to another distant planet w.r.t to an observer on Earth.

$$v = \frac{d}{t}$$



$$\sqrt{V_{\text{Vehicle}}} = \frac{L}{t_{\text{Vehicle}}} = \frac{L}{t_0}$$

$V_{\text{Vehicle}}$  - speed of spaceship  
 $t_{\text{Vehicle}}$  - time it takes to reach the other planet  
 $L$  - distance as measured

We use to notation based on last class from the vehicle

**Time dilation**

$$\sqrt{\frac{L_0}{t_E}} = \frac{L}{t_0} = V_V$$

$$\frac{L}{t_0} = \frac{L_0}{t_E}$$

but  $t_E = \frac{L_0}{\sqrt{1 - \frac{v^2}{c^2}}}$

$$\Rightarrow \frac{L}{L_0} = \frac{\frac{L_0}{t_0}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\Rightarrow \frac{L}{L_0} = \frac{L_0}{1} \cdot \sqrt{1 - \frac{v^2}{c^2}}$$

$$\cancel{L_0} \frac{L}{\cancel{L_0}} = \left[ \frac{L_0 \sqrt{1 - \frac{v^2}{c^2}}}{t_0} \right] \cancel{t_0}$$

→  $L = L_0 \sqrt{1 - \frac{v^2}{c^2}}$

$L$  = length experienced  
by object traveling  
very fast

$L_0$  = proper length  
length experienced by  
observer on earth

$$L < L_0 \text{ always}$$

\* length contraction is only in the direction of motion

ex) a space vehicle that is 50.0m long is  
travelling @ a constant speed of .75c past an  
observer standing on earth. How long does it appear  
to an observer on Earth?

$$L = L_0 \sqrt{1 - \frac{v^2}{c^2}}$$

$$= 50.0 \sqrt{1 - \left(\frac{.75c}{c}\right)^2}$$

$$= 33 \text{ m}$$

ex) a space vehicle flies parallel to a horizontal meter stick. The meter stick appears to be only 0.50m long. How fast was the ship travelling?

$$L = L_0 \sqrt{1 - \frac{v^2}{c^2}}$$

$$\left(\frac{L}{L_0}\right)^2 \cdot 1 - \frac{v^2}{c^2} \Rightarrow c^2 \boxed{1 - \left(\frac{L}{L_0}\right)^2 = \frac{v^2}{c^2}}$$

$$c^2 \left[1 - \left(\frac{.5}{1}\right)^2\right] = \frac{v^2}{c^2}$$

$$v = \underline{2.6 \times 10^8 \text{ m/s}}$$

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Mass Increase

$$\sqrt{c^2 \cdot \left(1 - \left(\frac{L}{L_0}\right)^2\right)} = v$$

$$\sqrt{c^2 (1 - .25)} = v$$

$$\sqrt{.75 c^2} = v$$

$$.87 c = v$$

The same way time and distance depends on the frame of reference , mass also depends on the frame of reference . The mass of an object increases with speed according to the following relativity equation

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$m$  = relativistic mass  
 $m_0$  = mass @ rest  
 $v$  = speed of the object  
 $c$  = speed of light.

$$m > m_0$$

Relativistic mass is always greater than rest mass

Another consequence of Einstein's special theory of relativity is a relationship between

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Another consequence of Einstein's special theory of relativity is a relationship between

$$E = mc^2$$

$E$  = energy

$m$  = mass of object.

$c$  = speed of light.

This says that mass can be transformed into energy in fact, that mass and energy are the equivalent

ex) Calculate the relativistic mass of an electron that is travelling  $2.0 \times 10^8 \text{ m/s}$ .

We know that the relativistic mass is always greater than mass @ rest.

$$\text{Mass of electron} = 9.11 \times 10^{-31} \text{ kg}$$

$$\Rightarrow m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$v = 2.0 \times 10^8 \text{ m/s}$$

$$\Rightarrow m = \frac{9.11 \times 10^{-31} \text{ kg}}{\sqrt{1 - \left(\frac{2.0 \times 10^8 \text{ m/s}}{3.00 \times 10^8 \text{ m/s}}\right)^2}} = 1.2 \times 10^{-30} \text{ kg}$$

ex2) What mass would be required to produce  $2.0 \times 10^{15} \text{ J}$  of heat if the mass was converted to heat.

$$\underline{E = mc^2}$$

$$E = 2.0 \times 10^{15} \text{ J}$$

$$c = 3.00 \times 10^8 \text{ m/s}$$

$$m = \frac{E}{c^2} = \frac{2.0 \times 10^{15} \text{ J}}{(3.00 \times 10^8 \text{ m/s})^2} = .022 \text{ kg}$$

$$\text{Recall efficiency} = \frac{\text{Workout}}{\text{Work in}} \times 100\%$$

ex 3 let's assume we have a machine that is 90% efficient in converting matter into energy. How much matter would we need to convert into  $7.0 \times 10^7 \text{ J}$

$$90\% = \frac{\text{Workout}}{\text{Workin}} \Rightarrow \text{Workin} = \frac{\text{Workout}}{90\%}$$

$$\Rightarrow \frac{7.0 \times 10^7 \text{ J}}{.9} = 77777777.78 \text{ J}$$

$$7.7 \times 10^7 \text{ J} = mc^2$$

$$m = \frac{7.7 \times 10^7 \text{ J}}{c^2} = 8.6 \times 10^{-10} \text{ kg}$$

ex 4) A .046 kg golf ball is lying on the green. If it was converted 100% into energy and this energy was used to run a 75 W bulb how long (yrs) would the light bulb be on?

$$E = mc^2$$

$$= (0.046 \text{ kJ}) (3.00 \times 10^8 \text{ J/kJ})^2$$

$$= 4.1 \times 10^{15} \text{ J}$$

$$P = \frac{\omega}{t} = \frac{E}{t} \Rightarrow t = \frac{E}{P} = \frac{4.1 \times 10^{15} \text{ J}}{75 \text{ W}}$$

$$= 5.5 \times 10^{13} \text{ s}$$

$$5.5 \times 10^{13} \times \frac{1 \text{ min}}{60 \text{ s}} \times \frac{1 \text{ hr}}{60 \text{ min}} \times \frac{1 \text{ day}}{24 \text{ hrs}} \times \frac{1 \text{ yr}}{365 \text{ days}}$$

$$t = 1.7 \times 10^6 \text{ yr} \quad \text{or 1.7 million years}$$

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Rd [pg 381] before class  
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$$m = \underline{m_0}$$

$$\sqrt{1 - \frac{v^2}{c^2}}$$

$$3m_0 = m_0 \overline{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\left( \sqrt{1 - \frac{v^2}{c^2}} \right)^2 = \frac{m_0}{3m_0} = \left( \frac{1}{3} \right)^2$$

$$1 - \frac{v^2}{c^2} = \frac{1}{9}$$

$$1 - \frac{1}{9} = \frac{v^2}{c^2}$$

$$\frac{8}{9} c^2 = v^2$$

$$v = \frac{2\sqrt{2}}{3} c \\ = .94c$$